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463. Proposed by H. O. HANSON, East Elmhurst, N. Y.

Find the sum of the series

$$\binom{2n}{0}+2\binom{2n-1}{1}+2^2\binom{2n-2}{2}+\cdots+2^n\binom{n}{n},$$

where $\binom{n}{r}$ denotes the coefficient of x^r in the expansion of $(1+x)^n$.

SOLUTION BY EDWIN R. SMITH, State College, Pa.

Consider the expansion of $1/(1-x-2x^2)$ into a series of ascending powers of x. First,

$$\frac{1}{1-x-2x^2}=\frac{1}{1-(x+2x^2)}=1+(x+2x^2)+(x+2x^2)^2+\cdots$$
$$=1+\binom{1}{0}x+\left[\binom{2}{0}+\binom{1}{1}2\right]x^2+\left[\binom{3}{0}+\binom{2}{1}2\right]x^3+\cdots$$
$$+\left[\binom{2n}{0}+2\binom{2n-1}{1}+2^2\binom{2n-2}{2}+\cdots+2^n\binom{n}{n}\right]x^{2n}+\cdots.$$

A second expansion can be obtained as follows:

$$\frac{1}{1-x-2x^2}=\frac{1}{1+x}\cdot\frac{1}{1-2x}$$
$$=(1-x+x^2-x^3+\cdots)(1+2x+2^2x^2+2^3x^3+\cdots)$$
$$=1-(1-2)x+(1-2+2^2)x^2-(1-2+2^2-2^3)x^3+\cdots$$
$$+(1-2+2^2-2^3+\cdots+2^{2n})x^{2n}-\cdots$$
$$=1-\frac{1-2^2}{1+2}x+\frac{1+2^3}{1+2}x^2-\cdots+\frac{1+2^{2n+1}}{1+2}x^{2n}-\cdots.$$

Comparing in the two series the expressions for the coefficient of x^{2n} there is obtained

$$\binom{2n}{0}+2\binom{2n-1}{1}+2^2\binom{2n-2}{2}+\cdots+2^n\binom{n}{n}=\frac{1+2^{2n+1}}{1+2}=\tfrac{1}{3}(2^{2n+1}+1),$$

which is the required sum.

Also solved by A. M. KENYON, N. P. PANDYA, and the PROPOSER.

464. Proposed by GEORGE Y. SOSNOW, Newark, N. J.

Find the general term and the sum of n terms of the series 1, 4, 15, 56, $\cdots$, where

$$U_n=4U_{n-1}-U_{n-2}.$$

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Changing the notation so that for U_n we may write U_{n+2}, we have

$$U_{n+2}-4U_{n+1}+U_n=0,$$

an equation in finite differences. Integrating, noticing that the roots of

$$m^2-4m+1=0,$$

are

$$m_1=2+\sqrt{3},$$
$$m_2=2-\sqrt{3},$$

(1) $$U_n=C_1(2+\sqrt{3})^n+C_2(2-\sqrt{3})^n.$$